TITLE: Resonant Coupling of Cavity-Backed Slots

# RESONANT COUPLING OF CAVITY-BACKED SLOTS

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## ABSTRACT

A rigorous solution of the problem of a TE wave scattering by a finite number of slots in a PEC plane backed by resonant cavities is given. The scattering problem formulated in terms of system of singular integral equations has been solved by the method of analytical regularization. Application of the regularization procedure allowed obtaining an accurate and numerically efficient solution.

## INTRODUCTION

The problem of scattering of a TE wave by a slot in a PEC plane, backed by a resonant cavity, or by a trough in a plane, has been discussed by many authors [1]-[2] in the context of the method of moments and the mode matching method. A numerically efficient solution of the problem, which works well on resonant and non-resonant frequencies, can be obtained, if a procedure of regularization is applied to an initial integral equation or a system of linear algebraic equations. As, there is no a "standard" procedure of regularization [3], for each problem a regularization algorithm, if exists, must be derived independently. The regularization algorithm employed in this paper originally was designed for elasto-dynamic problems and described in [4]. Lately it was used in different scattering problems [5], [6]. Application of the regularization algorithm [4] to the problem of a TE wave scattering by a cavity backed slot allows the transformation of initial integral equation into infinite system of linear algebraic equations. As the system of equations is well conditioned its solution can be found numerically, with desired accuracy, from a truncated system of equations. In the case of a narrow slot, slot width doesn't exceed one tenth of wavelength, the solution of the problem can be obtained in an analytical form. An efficient solution of the single slot problem provides a possibility to obtain solutions for more complicated structures, such as a collection of a finite number of cavity-backed slots, with the same high accuracy and numerical efficiency.

## PROBLEM FORMULATION

Consider N cavity backed slots as shown in Fig.1. The problem of a TE wave scattering by slots loaded on resonant cavities is formulated by using Green function formalism. Green functions of the upper half-space and of a rectangular cavity for the Neuman boundary condition under assumption that $y = y' = 0$ are defined as follows:

$$G_{H}(x,x') = -\frac{k}{2W} H_0^{(2)}(k|x-x'|), \qquad (1)$$

$$G_m(x,x') = -\frac{ik}{2Wa_m}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{\gamma_n^m}\coth(\gamma_n^m b)\cos(\frac{n\pi}{2a_m}(x+d_m))\cos(\frac{n\pi}{2a_m}(x'+d_m)) \qquad (2)$$

where $k = {2\pi}/{\lambda}$ , $H_0^{(2)}(z)$ - Hankel function of zero order of the second kind, W-free space impedance, $\varepsilon_m = 1$, $m \neq 0$, $\varepsilon_0 = 2$, $\gamma_n^m = \sqrt{(\frac{n\pi}{2a_m})^2 + k^2}$ . The subscript m in (2) corresponds to the cavity number. The cavity parameters - $a_m, b_m, c_m$, and $d_m$ are defined as shown in the Fig.2, where $2a_m$ - cavity width in the X direction, $b_m$ - cavity depth, $2c_m$ - slot width and $d_m$ -defines slot position with respect to the upper left corner

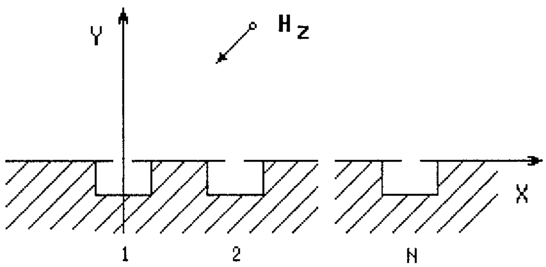


Fig.1 The original problem.

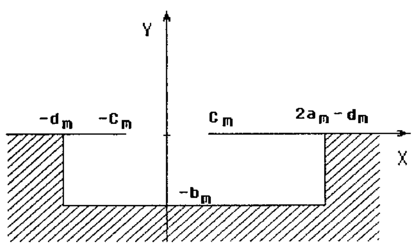


Fig.2 Geometry of the cavity-backed slot.

of the cavity ( $d_m = a_m$ corresponds to the case when slot center is positioned at the mid point of the interval $[-a_m, a_m]$ ). If the system of N slots is exited by a TE wave, a system of N integral equations for the problem can be formulated as

$$\int_{-C_m}^{+C_m} E_m(x')[G_H(x,x') + G_m(x,x')]dx' = -2f(x) - 2\sum_{\substack{i=1 \\ i \neq m}}^{N} H_i(x) , \qquad (3)$$

where $E_m(x)$ - is an unknown magnetic current on the slot m, $f(x)$ - the incident field, $H_i(x)$ - is a secondary incident field produced by the magnetic current on the slot i. The system of singular integral equations (3) can be reduced, by the method of analytical regularization, to a system of linear algebraic equations of the form

$$z_m^k A_m^k + \sum_{l=0}^{M} z_l^k B_{m,l}^k + \sum_{\substack{i=1 \\ i \neq k}}^{N} \sum_{l=0}^{M} z_l^i D_{m,l}^i = f_m^k , \qquad (4)$$

where $Z_m^i$ are unknown Tchebyshev coefficients of magnetic current on the slot i. The system of equations (4) allows us to calculate M unknown coefficients in the unknown magnetic current expansion for each slot, once these coefficients are calculated the scattered field is determined at any point in the upper half-space and inside resonant cavities.

## NUMERICAL RESULTS

The far field diagram of N slots is defined as

$$H = \sum_{n=1}^{N} kc_n e^{ikx_n \cos(\varphi)} \sum_{j=0}^{M} z_j^n \int_{-1}^{1} \frac{e^{ikc_n \cos(\varphi) x} T_j(x) dx}{\sqrt{1-x^2}}, \tag{5}$$

where the polar angle $\varphi$ is counted counterclockwise of the OX axis. Fig.3,4,5 presents graphs of the scattered field at the point $\varphi = \pi/2$, versus the frequency parameter ka calculated by the formula (5). The incident field is a plane wave normally incident on slots. Fig.3 presents scattered field of a single slot, and Fig.4,5 present graphs of scattered field by two slots. In the case of two slots both of them have the same slot width and cavity size, here $a = a_1 = a_2$. Fig.3 presents graph of $|H|$ for a single slot with the following cavity parameters $b_1 = 2a_1, c_1 = 0.2a_1, d_1 = a_1$. Maximums in the scattered field occurs at resonant frequencies of cavity, coupled through the slot with the upper half-space. Fig, 4 presents graph of scattered field by two slots for the case of an optimum coupling between slots on the $H_{00}$ mode, the distance between slots is defined by the formula $kx_2 = 2ka + 0.95$. Fig.5 shows the effect of the resonant coupling between slots on the $TE_{10}$ mode, in this case the distance between slots is defined by $kx_2 = 2ka + 2.5$. Presented numerical results demonstrates, that the mutual coupling effect can drastically change the scattering properties of cavity-backed slots at resonant frequencies

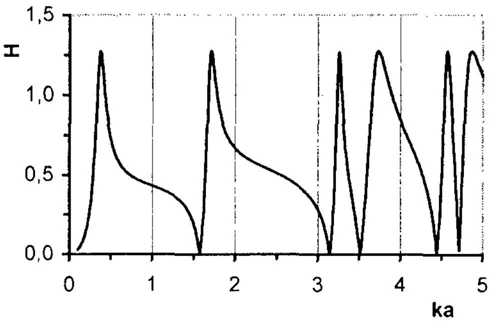


Fig. 3 Scattered field at $\varphi = \pi/2$, one slot, kb=2ka, kc=0.2ka, kd=ka.

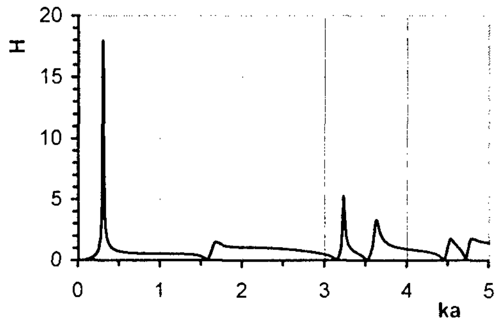


Fig.4 The effect of the resonance coupling on the $H_{00}$ mode.

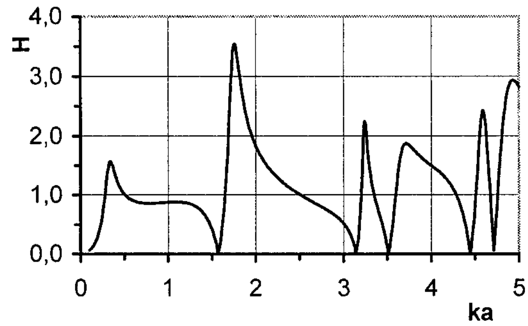


Fig. 5 The effect of resonance coupling on the $TE_{10}$ mode

## CONCLUSSION

In this paper the exact solution of the problem of a TE wave scattering by a collection of finite number of cavity- backed slots is given. The method automatically includes the effect of singular behavior of a tangent component of the electric field on slot aperture. The solution has no limitations on cavity size and is numerically efficient. The examples presented in the paper demonstrate the coupling effect on the scattering properties of cavity- backed slots. It has been shown that the method works with the same efficiency on resonant and non-resonant frequencies.

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